CSE 5526: Introduction to Neural Networks

SVM and WTA in-class problems

SVM Problem 1

- Demonstrate that the RBF, polynomial, and tanh kernels satisfy
$$k(\boldsymbol{x}, \boldsymbol{x}') = k(Q\boldsymbol{x}, Q\boldsymbol{x}')$$
  - For a matrix Q that is unitary, i.e., $Q^{-1} = Q^T$

- Does this property hold for the following kernel?
$$k(\boldsymbol{x}, \boldsymbol{x}') = \boldsymbol{x}^T A \boldsymbol{x}'$$
  - where A is a symmetric and positive semidefinite matrix
  - EC: prove that this is a valid kernel

SVM Problem 2

- Show that Mercer kernels satisfy the Cauchy-Schwarz inequality

$$k(\boldsymbol{x}, \boldsymbol{x}')k(\boldsymbol{x}', \boldsymbol{x}) \leq k(\boldsymbol{x}, \boldsymbol{x})k(\boldsymbol{x}', \boldsymbol{x}')$$

Hint: use the determinant of a 2×2 Gram matrix

WTA Problem 1

- Consider a winner-take-all network with 5 neurons, the function of each neuron is defined as

$$y_i(t+1) = \varphi\left((S-1)y_i(t) - \sum_{j \neq i} y_j(t)\right)$$

  - where S is the number of neurons, and

$$\varphi(x) = \begin{cases} 0, & x \leq 0 \\ x, & 0 < x \leq 1 \\ 1, & 1 < x \end{cases}$$

- Find the network output at time steps 1, 2, and ∞
  - For the input $\boldsymbol{y}(0) = [1.0, 0.9, 0.0, 0.1, 0.5]^T$
  - For the input $\boldsymbol{y}(0) = [0.2, 0.1, 0.0, 0.1, 0.2]^T$

WTA Problem 2

- Consider a winner-take-all network with 5 neurons, the function of each neuron is defined as

$$y_i(t+1) = \varphi\left(a y_i(t) - b \sum_{j \neq i} y_j(t)\right)$$

  - where $\varphi(x) = \begin{cases} 0, & x \leq 0 \\ x, & 0 < x \leq 1 \\ 1, & 1 < x \end{cases}$

- Find values or ranges of a and b such that
  - $y_i(\infty) = 1$ for the maximum $y_i(0)$
  - $y_i(\infty) = 0$ for all other $y_i(0)$
  - When $0 \leq y_i(0) \leq 1$ for all i